Missouri Mathematics Core Academic Standards

Shift One: Focus

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Missouri
DEPARTMENT OF ELEMENTARY & SECONDARY
EDUCATION™

Session Overview

- Shift One
- Rationale for Shift One
- Curriculum Implications
- Resources
- Additional Information

Core Academic Standards (CAS) Mathematics Shifts

Shift 1: **FOCUS**

Shift 2: **COHERENCE**

Shift 3: **RIGOR**

- **Conceptual Understanding**
- **Fluency**
- **Applications**

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Mathematics CAS Shift One: **FOCUS**

The CAS significantly *narrow* the scope of content and **deepen** how time and energy is spent in the classroom.

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Traditional Approach to U.S. Mathematics Curriculum

GRADES									
K	1	2	3	4	5	6	7	8	9 - 12
Number and Operations									
Algebra Relationships									
Geometry									
Measurement									
Data Analysis and Probability									

"A mile wide and inch deep curriculum…"

Composite of high achieving countries

Note that topics are introduced and sustained in a coherent fashion, producing a clear upper-triangular structure.

TOPIC	GRADE: 1	2	3	4	5	6	7	8
Whole Number Meaning	■	■	■	▒	▒			
Whole Number Operations	■	■	■	■	▒			
Measurement Units	□	■	■	■	■	■	▒	
Common Fractions			□	■	■	▒		
Equations & Formulas			□	▒	▒	▒	■	■
Data Representation & Analysis			□	□	▒	▒		□
2-D Geometry: Basics			□	▒	▒	▒	■	■
Polygons & Circles				▒	▒	▒	■	■
Perimeter, Area & Volume				▒	▒	▒	▒	□
Rounding & Significant Figures				▒	▒			
Estimating Computations				▒	▒	▒		
Properties of Whole Number Operations				□	▒			
Estimating Quantity & Size				□	□			
Decimal Fractions				▒	■	▒		
Relationship of Common & Decimal Fractions				▒	■	▒		
Properties of Common & Decimal Fractions					▒	▒		
Percentages					▒	▒		
Proportionality Concepts					▒	▒	▒	□
Proportionality Problems					▒	▒	■	■
2-D Coordinate Geometry					□	□	▒	▒
Geometry: Transformations						▒	▒	▒
Negative Numbers, Integers & Their Properties						□	▒	
Number Theory							▒	□
Exponents, Roots & Radicals							▒	▒
Exponents & Orders of Magnitude							□	□
Measurement Estimation & Errors							□	
Constructions w/ Straightedge & Compass							■	□
3-D Geometry							▒	■
Congruence & Similarity								■
Rational Numbers & Their Properties								□
Patterns, Relations & Functions								□
Slope & Trigonometry								□
Number of topics covered by at least 67% of the A+ countries	3	3	7	15	20	17	16	18
Number of additional topics intended by A+ countries to complete a typical curriculum at each grade level	2	6	5	1	1	3	6	3

□ – intended by 67% of the A+ countries ▒ – intended by 83% of the A+ countries ■ – intended by 100% of the A+ countries

- Mathematics topics intended at each grade by at least two thirds of A+ countries.
- A+ countries determined by looking at statistically significant differences in 8th grade scores on 1995 TIMSS
- On average an A+ country would have 1–6 more topics per grade level in its complete curriculum.

Composite of U.S. State Curricula

Note that topics are introduced and sustained in a way that produces no visible structure.

TOPIC	GRADE: 1	2	3	4	5	6	7	8
Whole Number Meaning	■	■	■	■	▒	□		
Whole Number Operations	■	■	■	■	▒	□		
Measurement Units	▒	■	■	▒	■	■	■	▒
Common Fractions	□	▒	▒	▒	■	▒	□	□
Equations & Formulas	□	□	▒	▒	▒	■	■	■
Data Representation & Analysis	■	■	■	■	■	■	■	▒
2-D Geometry: Basics	▒	▒	▒	▒	▒	▒	▒	▒
Polygons & Circles	■	■	■	■	■	■	■	▒
Perimeter, Area & Volume		□	□	□	▒	■	■	▒
Rounding & Significant Figures								
Estimating Computations	□	□	▒	▒	▒	▒	▒	▒
Properties of Whole Number Operations	□	□	□	□				
Estimating Quantity & Size			□					
Decimal Fractions			□	▒	▒	▒	□	□
Relationship of Common & Decimal Fractions				□	□	□		
Properties of Common & Decimal Fractions								
Percentages					□	▒	▒	□
Proportionality Concepts						▒	□	
Proportionality Problems						▒	▒	▒
2-D Coordinate Geometry			□	▒	□	□	□	▒
Geometry: Transformations	▒	▒	▒	▒	▒	▒	▒	▒
Negative Numbers, Integers & Their Properties						□	▒	□
Number Theory					▒	□	□	□
Exponents, Roots & Radicals						□	□	■
Exponents & Orders of Magnitude							□	□
Measurement Estimation & Errors	□	□	▒	□	▒	▒	▒	□
Constructions w/ Straightedge & Compass								
3-D Geometry	■	■	■	▒	■	▒	■	▒
Congruence & Similarity					□	▒	▒	□
Rational Numbers & Their Properties						▒	▒	□
Patterns, Relations & Functions	▒	■	■	■	▒	■	■	■
Slope & Trigonometry								
Number of topics covered by at least 67% of the states	14	15	18	18	20	25	23	22
Number of additional topics intended by states to complete a typical curriculum at each grade level	8	8	7	8	8	5	6	6

□ – intended by 67% of the states ▒ – intended by 83% of the states ■ – intended by 100% of the states

- Mathematics topics intended at each grade by at least two thirds of 21 U.S. States.
- On average a state would have 6–8 more topics per grade level in its complete curriculum.
- From Schmidt, Houang, and Cogan, *American Educator*, 2005.

Doing More of Less

One of the characteristics of the most effective schools is their willingness to declare that some things are more important than others; they are willing to abandon some less important content so as to be able to have enough time dedicated to those areas that are valued most.

Lezotte, L. W. (1991). Correlates of effective schools: The first and second generation. Okemos, MI: Effective School Products.

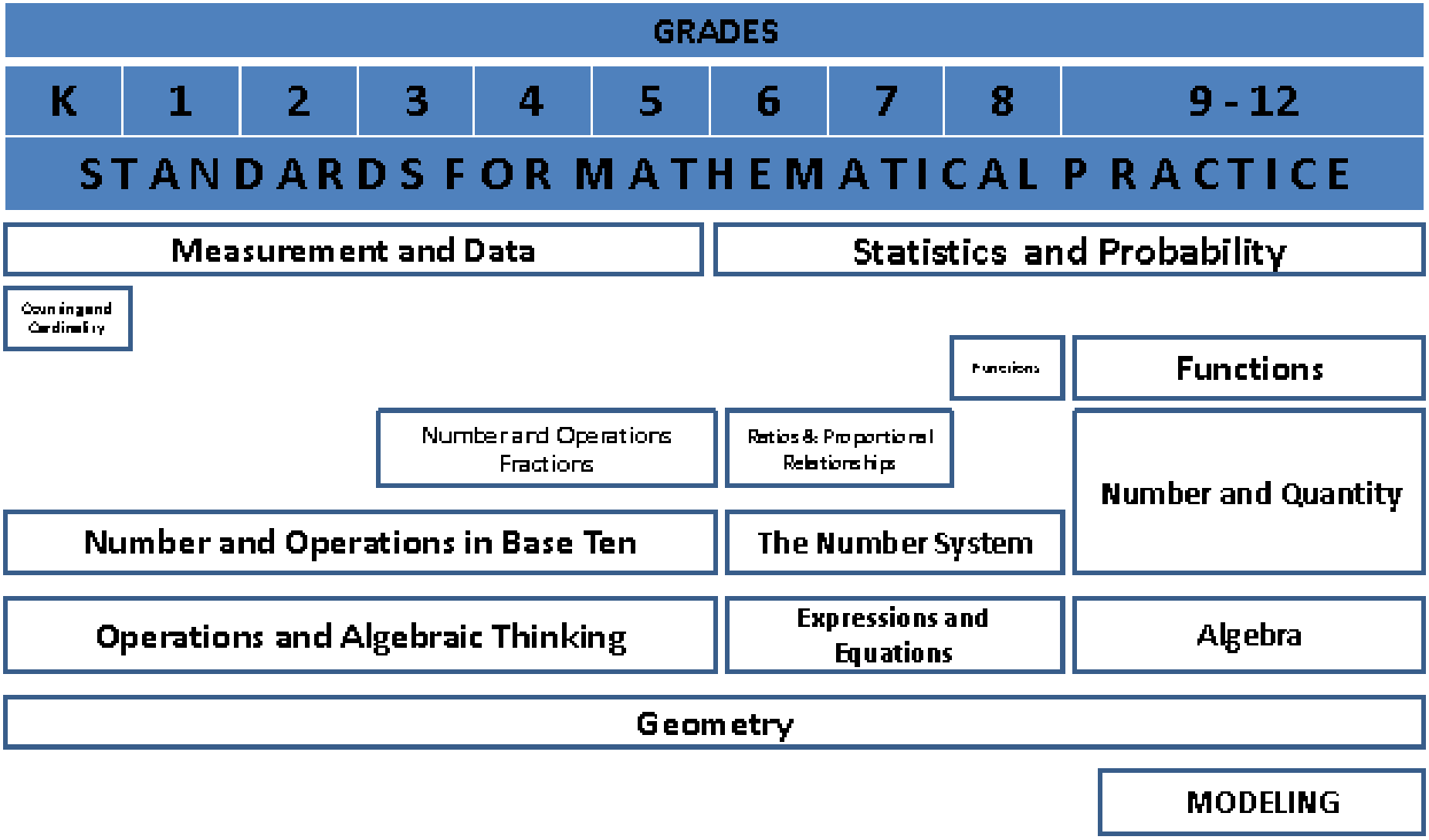
National Mathematics Advisory Panel

In 2008, the National Mathematics Advisory Panel recommended that mathematics curricular content should be a focused, coherent progression of mathematics learning, with an emphasis on proficiency with key topics.

Foundations for Success The Final Report of the National Mathematics Advisory Panel
http://www2.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf

Missouri Core Academic Standards in Mathematics

Domains and Conceptual Categories K - 12



Comparison of CCSS with A+ composite

Topic	1	2	3	4	5	6	7	8
Whole Number: Meaning	●	●	●	●	●			
Whole Number: Operations	●	●	●	●	●			
Measurement Units	●	●	●	●	●	●	●	
Common Fractions	●	●	●	●	●	●		
Equations & Formulas				●	●	●	●	●
Data Representation & Analysis	●	●	●	●	●	●	●	●
2-D Geometry: Basics		●	●	●	●	●	●	●
2-D Geometry: Polygons & Circles	●	●	●	●	●	●	●	●
Measurement: Perimeter, Area & Volume		●	●	●	●	●	●	●
Rounding & Significant Figures				●	●			
Estimating Computations				●		●		●
Whole Numbers: Properties of Operations	●	●	●	●	●			
Estimating Quantity & Size								
Decimal Fractions				●	●	●		
Relation of Common & Decimal Fractions			●	●	●	●		
Properties of Common & Decimal Fractions					●	●		
Percentages							●	
Proportionality Concepts						●	●	●
Proportionality Problems						●	●	●
2-D Geometry: Coordinate Geometry					●	●	●	●
Geometry: Transformations				●				●
Negative Numbers, Integers, & Their Properties						●	●	
Number Theory				●				
Exponents, Roots & Radicals						●		
Exponents & Orders of Magnitude								
Measurement: Estimation & Errors								
Constructions Using Straightedge & Compass							●	●
3-D Geometry	●	●			●	●	●	●
Geometry: Congruence & Similarity								●
Rational Numbers & Their Properties						●	●	●
Patterns, Relations & Functions								●
Proportionality: Slope & Trigonometry								●

● Topic Intended in Common Core Standards

Intended in More Than Half of Top Achieving Countries

- The number of extra topics per grade level in CCSS is comparable with A+ countries.

All grades, K – 12, should focus on the Standards for Mathematical Practice (1) Make sense of problems and persevere in solving them. (2) Reason abstractly and quantitatively. (3) Construct viable arguments and critique the reasoning of others. (4) Model with mathematics. (5) Use appropriate tools strategically. (6) Attend to precision (7) Look for and make use of structure. (8) Look for and express regularity in reasoning. COMMON CORE STATE STANDARDS INITIATIVE	In KINDERGARTEN instructional time should focus on two critical areas: (1) representing, relating, and operating on whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.	In GRADE 1, instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating units; and (4) reasoning about attributes of, and composing decomposing geometric shapes.	In GRADE 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.	In GRADE 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.
In GRADE 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties; such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.	In GRADE 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; (3) developing understanding of volume.	In GRADE 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.	In GRADE 7, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and – three dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.	In GRADE 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and –three dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

http://www.corestandards.org/

All grades, K – 12, should focus on the Standards for Mathematical Practice (1) Make sense of problems and persevere in solving them. (2) Reason abstractly and quantitatively. (3) Construct viable arguments and critique the reasoning of others. (4) Model with mathematics (5) Use appropriate t... strategically. (6) Attend to precision (7) Look for and make use of structure. (8) Look for and express regularity in reasoning. COMMON CORE STATE STANDARDS INITIATIVE	In KINDERGARTEN instructional time should f... areas: (1) rep... and... nu... of... (2) de... spe...	In GRADE 1, instructional time ... (4) reasoning about attributes of, and composing decomposing geometric shapes.	In GRADE 2, instructional time	In GRADE 3, instructional time ...critical areas: ...understanding ...tion and ...strategies for ...n and division ...understanding ...especially ...s (fractions ...ator 1); ...understanding ...ture of ...arrays and of ... (4) describing and analyzing two-dimensional shapes.
In GRADE 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties; such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.	In GRADE 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; (3) developing understanding of volume.	In GRADE 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.	In GRADE 7, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and – three dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.	In GRADE 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and –three dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

Mathematical Practice Standards to be include in all grades K – 12.

http://www.corestandards.org/

ALGEBRA I

Critical Area 1:

- Students analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems.
- They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations.

Critical Area 2:

- Students learn function notation and develop the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations.
- Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change.
- Students explore systems of equations and inequalities, and they find and interpret their solutions.
- They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Critical Area 3:

- Students experience more formal means of assessing how a model fits data.
- Students use regression techniques to describe approximate linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Critical Area 4:

- Students build on their knowledge to extended the laws of exponents to rational exponents.
- Students apply this new understanding of number and strengthen their ability to see structure in and create quadratic and exponential expressions.
- They create and solve equations, inequalities, and systems of equations involving quadratic expressions.

Critical Area 5:

- Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena.
- Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function.

  Students expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

GEOMETRY

Critical Area 1:

- Students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions.
- They use triangle congruence as a familiar foundation for the development of formal proof.
- Students prove theorems—using a variety of formats—and solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

Critical Area 2:

- Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity.
- They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean theorem.
- Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles, building on students' work with quadratic equations done in the first course.
- They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles.

Critical Area 3:

- Students' experience with two-dimensional and three-dimensional objects is extended to include informal explanations of circumference, area and volume formulas.
- Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line.

Critical Area 4:

- Building on their work with the Pythagorean theorem in 8th grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines, which relates back to work done in the first course.
- Students continue their study of quadratics by connecting the geometric and algebraic definitions of the parabola.

Critical Area 5:

- Students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures.
- They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center.
- Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations, which relates back to work done in the first course, to determine intersections between lines and circles or parabolas and between two circles.

Critical Area 6:

- Students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability.
- Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

ALGEBRA II

Critical Area 1:

- Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property.
- Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers.
- Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations.
- Students understand that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

Critical Area 2:

- Students now use the coordinate plane to extend trigonometry to model periodic phenomena.

Critical Area 3:

- Students synthesize and generalize what they have learned about a variety of function families.
- They extend their work with exponential functions to include solving exponential equations with logarithms.
- They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function.
- They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as "*the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions*" is at the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

Critical Area 4:

- Students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions.
- They identify different ways of collecting data— including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

COMMON CORE STATE STANDARDS INITIATIVE

All grades, K – 12, should focus on the Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision
7. Look for and make use of structure.
8. Look for and express regularity in reasoning.

http://www.corestandards.org/ (Adapted from ***Appendix A: Designing High School Mathematics Courses Based on the Common Core State Standards***)

ALGEBRA I

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Critical Area 2:

- Students learn function notation and develop the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations.
- Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change.
- Students explore systems of equations and inequalities, and they find and interpret their solutions.
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Critical Area 3:

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Critical Area 6:

- Students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability.
- Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

ALGEBRA II

Critical Area 1:

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- Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations.
- Students understand that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

Critical Area 2:

- Students now use the coordinate plane to extend trigonometry to model periodic phenomena.

Critical Area 3:

- Students synthesize and generalize what they have learned about a variety of function families.
- They extend their work with exponential functions to include solving exponential equations with logarithms.
- They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function.
- They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as "*the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions*" is at the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

Critical Area 4:

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- They identify different ways of collecting data— including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

COMMON CORE STATE STANDARDS INITIATIVE

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4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in reasoning.

Mathematical Practice Standards to be include in all grades K – 12.

http://www.corestandards.org/ (Adapted from ***Appendix A: Designing High School Mathematics Courses Based on the Common Core State Standards***)

CCSS Mathematics K - 8 Domains Emphases & Shifts 9/12

Domain	K	1	2	3	4	5	6	7	8
Counting and Cardinality	K.CC.1, 2, 3, 4, 4a, 4b, 4c, 5, 6, 7								
Operations and Algebraic Thinking	K.OA.1, 2, 3, 4, 5	1.OA.1, 2, 3, 4, 5, 6, 7, 8	2.OA.1, 2, 3, 4	3.OA.1, 2, 3, 4, 5, 6, 7, 8, 9	4.OA.1, 2, 3, 4, 5	5.OA.1, 2, 3			
Numbers and Operations in Base Ten	K.NBT.1	1.NBT.1, 2, 2a, 2b, 2c, 3, 4, 5, 6	2.NBT.1, 1a, 1b, 2, 3, 4, 5, 6, 7, 8, 9	3.NBT.1, 2, 3	4.NBT.1, 2, 3, 4, 5, 6	5.NBT.1, 2, 3, 3a, 3b, 4, 5, 6, 7			
Numbers and Operations - Fractions				3.NF.1, 2, 2a, 2b, 3a, 3b, 3c, 3d	4.NF.1, 2, 3, 3a, 3b, 3c, 3d, 4a, 4b, 4c, 5, 6, 7	5.NF.1, 2, 3, 4, 4a, 4b, 5, 5a, 4b, 5a, 5b, 6, 7, 7a, 7b, 7c			
Measurement and Data	K.MD.1, 2, 3	1.MD.1, 2, 3, 4	2.MD.1, 2, 3, 4, 5, 6, 7, 8, 9, 10	3.MD.1,2, 3, 4, 5, 5a, 5b, 6, 7, 7a, 7b, 7c, 7d, 8	4.MD.1, 2, 3, 4, 5, 5a, 5b, 6, 7	5.MD.1, 2, 3, 3a, 3b, 4, 5, 5a, 5b, 5c			
Geometry	K.G.1, 2, 3, 4, 5, 6	1.G.1, 2, 3	2.G.1, 2, 3	3.G.1, 2	4.G.1, 2, 3	5.G1, 2, 3, 4	6.G.1, 2, 3, 4	7.G1, 2, 3, 4, 5, 6	8.G.1, 1a, 1b, 1c, 2, 3, 4, 5, 6, 7, 8, 9
Ratios and Proportional Relationships							6.RP.1, 2, 3, 3a, 3b, 3c, 3d	7.RP.1, 2, 2a, 2b, 2c, 2d, 3	
The Number System							6.NS.1, 2, 3, 4, 5, 6, 6a, 6b, 6c, 7, 7a, 7b, 7c, 7d, 8	7.NS.1, 1a, 1b, 1c, 1d, 2, 2a, 2b, 2c, 2d, 3	8.NS.1, 2,
Expressions and Equations							6.EE1, 2a, 2b, 2c, 3, 4, 5, 6, 7, 8, 9	7.EE.1, 2, 3, 4, 4a, 4b	8.EE.1, 2, 3, 4, 5 ,6, 7, 7a, 7b, 8, 8a, 8b, 8c
Statistics and Probability							6.SP.1, 2, 3, 4, 5, 5a, 5b, 5c, 5d	7.SP.1, 2, 3, 4, 5, 6, 7, 7a, 7b, 8, 8a, 8b, 8c	8.SP.1, 2, 3, 4
Functions									8.F.1, 2, 3, 4, 5

CCSS Mathematics K - 8 Domains Emphases & Shifts 9/12

Domain	K	1	2	3	4	5	6	7	8
Counting and Cardinality	K.CC.1, 2, 3, 4, 4a, 4b, 4c, 5, 6, 7			**3– 5 Fractions**					
Operations and Algebraic Thinking	K.OA.1, 2, 3, 4, 5	1.OA.1, 2, 3, 4, 5, 6, 7, 8	2.OA.1, 2, 3, 4	3.OA.1, 2, 3, 4, 5, 6, 7, 8, 9	4.OA.1, 2, 3, 4, 5	5.OA.1, 2, 3			
Numbers and Operations in Base Ten	K.NBT.1	1.NBT.1, 2, 2a, 2b, 2c, 3, 4, 5, 6	2.NBT.1, 1a, 1b, 2, 3, 4, 5, 6, 7, 8, 9	3.NBT.1, 2, 3	4.NBT.1, 2, 3, 4, 5, 6	5.NBT.1, 2, 3, 3a, 3b, 4, 5, 6, 7			
Numbers and Operations - Fractions				3.NF.1, 2, 2a, 2b, 3a, 3b, 3c, 3d	4.NF.1, 2, 3, 3a, 3b, 3c, 3d, 4a, 4b, 4c, 5, 6, 7	5.NF.1, 2, 3, 4, 4a, 4b, 5, 5a, 4b, 5a, 5b, 6, 7, 7a, 7b, 7c			
Measurement and Data	K.MD.1, 2, 3	1.MD.1, 2, 3, 4	2.MD.1, 2, 3, 4, 5, 6, 7, 8, 9, 10	3.MD.1,2, 3, 4, 5, 5a, 5b, 6, 7, 7a, 7b, 7c, 7d, 8	4.MD.1, 2, 3, 4, 5, 5a, 5b, 6, 7	5.MD.1, 2, 3, 3a, 3b, 4, 5, 5a, 5b, 5c	**6 – 7 Ratios & Proportional Relationships**		
Geometry	K.G.1, 2, 3, 4, 5, 6	1.G.1, 2, 3	2.G.1, 2, 3	3.G.1, 2	4.G.1, 2, 3	5.G1, 2, 3, 4	6.G.1, 2, 3, 4	7.G1, 2, 3, 4, 5, 6	8.G.1, 1a, 1b, 1c, 2, 3, 4, 5, 6, 7, 8, 9
Ratios and Proportional Relationships	**K – 2 Computation**						6.RP.1, 2, 3, 3a, 3b, 3c, 3d	7.RP.1, 2, 2a, 2b, 2c, 2d, 3	
The Number System							6.NS.1, 2, 3, 4, 5, 6, 6a, 6b, 6c, 7, 7a, 7b, 7c, 7d, 8	7.NS.1, 1a, 1b, 1c, 1d, 2, 2a, 2b, 2c, 2d, 3	8.NS.1, 2,
Expressions and Equations			**8 Systems of Equations**				6.EE1, 2a, 2b, 2c, 3, 4, 5, 6, 7, 8, 9	7.EE.1, 2, 3, 4, 4a, 4b	8.EE.1, 2, 3, 4, 5 ,6, 7, 7a, 7b, 8, 8a, 8b, 8c
Statistics and Probability							6.SP.1, 2, 3, 4, 5, 5a, 5b, 5c, 5d	7.SP.1, 2, 3, 4, 5, 6, 7, 7a, 7b, 8, 8a, 8b, 8c	8.SP.1, 2, 3, 4
Functions									8.F.1, 2, 3, 4, 5

Algebra in elementary school is about properties and relationships.

Grade	Operations and Algebraic Thinking	Number and Operations in Base Ten	Fractions
1	Understand and apply properties of operations and the relationships between addition and subtraction.	Use place value understanding and properties of operations to add and subtract.	
2		Use place value understanding and properties of operations to add and subtract.	
3	Understand properties of multiplication and the relationship between multiplication and division.	Use place value understanding and properties of operations to perform multi-digit arithmetic.	
4		Use place value understanding and properties of operations to perform multi-digit arithmetic.	Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
5			Apply and extend previous understandings of multiplication and divisions to multiply and divide fractions.

CCSS Mathematics Traditional High School Pathway Domains 9/12

Conceptual Category	Domain	Algebra I	Geometry	Algebra II	Fourth Course
Number and Quantity	The Real Number System	N.RN.1, 2, 3			
	Quantities	N.Q. 1, 2, 3			
	The Complex Number System			N.CN.1, 2, 7, (+)8, (+)9	(+)N.CN.3, 4, 5, 6
	Vector and Matrix Quantities				(+)N.VM.1, 2, 3, 4a, 4b, 4c, 5a, 5b, 6, 7, 8, 9, 10, 11, 12
Algebra	Seeing Structure in Expressions	A.SSE.1a, 1b, 2, 3a, 3b, 3c		A.SSE.1a, 1b, 2, 4	
	Arithmetic with Polynomials and Rational Expressions	A.APR.1		A.APR.1, 2, 3, 4 (+)5, 6, (+)7	
	Creating Equations	A.CED. 1, 2, 3, 4		A.CED.1, 2, 3, 4	
	Reasoning with Equations and Inequalities	A.REI.I, 3, 4a, 4b, 5, 6, 7, 10, 11, 12		A.REI.2, 11	(+) A.REI. 8, 9
Functions and Models	Interpreting Functions	F.IF.1, 2, 3, 4, 5, 6, 7a, 7b, 7e, 8a, 8b, 9		F.IF.4, 5, 6, 7b, 7c, 7e, 8, 9	F.IF.7d
	Building Functions	F.BF.1a, 1b, 2, 3, 4a		F.BF.1b, 3, 4a	(+)F.BF.1c, 4c, 4d, 5
	Linear, Quadratic and Exponential Models	F.LE.1a, 1b, 1c, 2, 3, 5		F.LE.4	
	Trigonometric Functions			F.TF.1, 2, 5, 8	(+)F.TF.3, 4, 6, 7, 9
Geometry	Congruence		G.CO.1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13		
	Similarity, Right Triangles, and Trigonometry		G.SRT.1a, 1b, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11		
	Circles		G.C. 1, 2, 3, (+)4, 5		
	Expressing Geometric Properties with Equations		G.GPE.1, 2, 4, 5, 6, 7		(+)G.GPE.3
	Geometric Measurement and Dimension		G.GMD.1, 3, 4		(+)G.GMD.2
	Modeling with Geometry		G.GMG.1, 2, 3		
Statistics and Probability	Interpreting Categorical and Quantitative Data	S.ID.1, 2, 3, 5, 6a, 6b, 6c, 7, 8, 9		S.ID.4	
	Making Inferences and Justifying Conclusions			S.IC.1, 2, 3, 4, 5, 6	
	Conditional Probabilities and the Rules of Probability		S.CP.1, 2, 3, 4, 5, 6, 7, (+)8, (+)9		
	Using Probability to Make Decisions		(+)S.MD.6, 7	(+)S.MD.6, 7	(+)S.MD.5a, 5b

CCSS Mathematics Traditional Pathway HS Domains 7/2011

Conceptual Category	Domain	Algebra I	Geometry	Algebra II	Fourth Course
Number and Quantity	The Real Number System	N.RN.1, 2, 3			
	Quantities	N.Q. 1, 2, 3			
	The Complex Number System			N.CN.1, 2, 7, (+)8, (+)9	(+)N.CN.3, 4, 5, 6
	Vector and Matrix Quantities				(+)N.VM.1, 2, 3, 4a, 4b, 4c, 5a, 5b, 6, 7, 8, 9, 10, 11, 12
Algebra	Seeing Structure in Expressions	A.SSE.1a, 1b, 2, 3a, 3b, 3c		A.SSE.1a, 1b, 2, 4	
	Arithmetic with Polynomials and Rational Expressions	A.APR.1		A.APR.1, 2, 3, 4 (+)5, 6, (+)7	
	Creating Equations	A.CED. 1, 2, 3, 4		A.CED.1, 2, 3, 4	
	Reasoning with Equations and Inequalities	A.REI.I, 3, 4a, 4b, 5, 6, 7, 10, 11, 12		A.REI.2, 11	(+) A.REI. 8, 9
Functions and Models	Interpreting Functions	F.IF.1, 2, 3, 4, 5, 6, 7a, 7b, 7e, 8a, 8b, 9		F.IF.4, 5, 6, 7b, 7c, 7e, 8, 9	F.IF.7d
	Building Functions	F.BF.1a, 1b, 2, 3, 4a		F.BF.1b, 3, 4a	(+)F.BF.1c, 4c, 4d, 5
	Linear, Quadratic and Exponential Models	F.LE.1a, 1b, 1c, 2, 3, 5		F.LE.4	
	Trigonometric Functions			F.TF.1, 2, 5, 8	(+)F.TF.3, 4, 6, 7, 9
Geometry	Congruence		G.CO.1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13		
	Similarity, Right Triangles, and Trigonometry		G.SRT.1a, 1b, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11		
	Circles		G.C. 1, 2, 3, (+)4, 5		
	Expressing Geometric Properties with Equations		G.GPE.1, 2, 4, 5, 6, 7		(+)G.GPE.3
	Geometric Measure				(+)G.GMD.2
	Modeling with Geo				
Statistics and Probability	Interpreting Catego				
	Making Inferences a				
	Conditional Probabi		(+)8, (+)9		
	Using Probability to Make Decisions		(+)S.MD.6, 7	(+)S.MD.6, 7	(+)S.MD.5a, 5b

★Modeling

MISSOURI MATHEMATICS CORE ACADEMIC STANDARDS CROSSWALK TO MISSOURI GLES/CLES
CONTENT ALIGNMENTS AND SHIFTS- Grade 5 *DRAFT*

http://dese.mo.gov/divimprove/curriculum/common-core-math.htm

Grade 5	
Critical Areas In Grade 5, instructional time should focus on three crit[illegible]eas: 1. developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cased (unit fractions divided by whole numbers and whole numbers divided by unit fractions); 2. extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and 3. developing understanding of volume.	**Mathematical Practices** 1. Make sense of problems and persevere in solving th 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Core Academic Standard (CAS) **Bold/Highlighted portions** of the CAS indicate content that does not align to any existing GLE/CLE for any course or grade. This content should be included in the instruction and assessment for Grade 5 upon transition to the mathematics CAS. ***Note: The link(s) provided from the Illustrative Mathematics Project in the CAS column provide draft examples intended to illustrate and clarify the CAS.***	**Grade 5 GLE** ***Bold, ITALICIZED portions*** of the 2008 Missouri GLE indicate content that aligns to the CAS for Grade 5. This content should be included in the instruction and assessment for Grade 5 upon transition to the mathematics CAS.	**GLE Shift to Grade 5** ***Bold, ITALICIZED portions*** of these off-grade 2008 Missouri GLEs indicate content that aligns to the CAS for Grade 5. This content should be included in the instruction and assessment for Grade 5 upon transition to the mathematics CAS.
Operations and Algebraic Thinking 5.OA		
Write and interpret numerical expressions.		
5.OA.1 Use parentheses, brackets, or braces i[illegible] numerical expressions, and ev[illegible] expressions with these symb[illegible] http://illustrativemathematics.org/illustrations/555	**A2B5** ***use the commutative, distributive and associative properties*** for fractions and decimals	**N2C6** ***apply properties of operations (including order of operations)*** to positive rational numbers

Implementing Mathematics CAS Shift One: **FOCUS**

- Focus deeply on what is emphasized in the Standards
- Turn loose of content/units that are not relevant or aligned to the Critical Focus Areas for a course or grade
- Evaluate textbooks for alignment to the Mathematics Core Academic Standards
- Revise curriculum and assessments aligned to the Critical Focus Areas and specific content identified for a grade/course
- Plan “in-depth” instruction
- Provide students opportunities to engage in the learning of mathematics through the inclusion of the Standards for Mathematical Practice

Resources

- DESE Mathematics Core Academic Standards Resources
http://dese.mo.gov/divimprove/curriculum/common-core-math.htm
- Common Core State Standards and Appendix A
http://www.corestandards.org/
- Illustrative Mathematics
http://commoncoretools.me/2011/01/16/the-illustrative-mathematics-project/
- National Council of Teachers of Mathematics
www.nctm.org
- *Foundations for Success: The Final Report of the National Mathematics Advisory Panel*
http://www2.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf

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